Université Bordeaux I
Mathématiques Appliquées de Bordeaux

ESF / FBP WORKSHOP

# FREE BOUNDARY PROBLEMS IN COMBUSTION

Arcachon, France
22-24 March 1995

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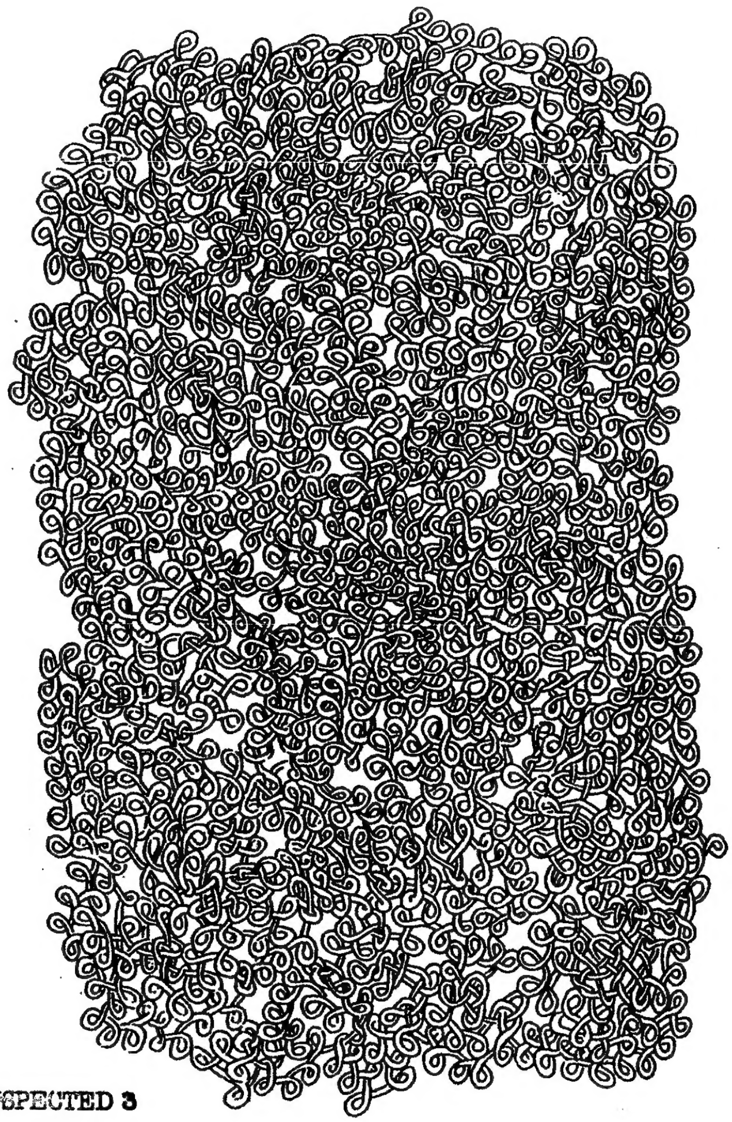
## Abstracts





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## ESF / FBP WORKSHOP

# FREE BOUNDARY PROBLEMS IN COMBUSTION

22-24 March 1995
Arcachon (France)

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# Regularity of the Free Boundary in Parabolic Phase Transition Problems

by
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## ABSTRACT

We discuss the regularity properties of the free boundary for parabolic two phase free boundary problems. The best example of such problems is the Stefan problem, a simplified model describing the melting of a material with a solid-liquid interphase. By assuming that the interphase is Lipschitz we encounter the fact that corners can persist for some time i.e. we do not have instantaneous smoothing. Nevertheless, if we consider non-degenerate cases or if assume that the Lipschitz constant is small, Lipschitz free boundaries in space and time are actually smooth. The method we present is an iterative scheme which relies heavily on the hyperbolic character of free boundary relation.

# Modélisation Mathématique de Flammes Planes en Chimie Complexe

par
A. Bonnet
Ecole Normale Supérieure
Paris, France

avec H. Berestycki et B. Larrouturou

## ABSTRACT

Les phénomènes de combustion font intervenir des dizaines d'espèces chimiques et des centaines de réactions. Dans le modèle thermodiffusif, la température du mélange et les concentrations des espèces sont solution d'un système d'équations de réaction-diffusion. On s'intéresse à l'existence et à l'unicité des ondes progressives pour ce problème.

# Two-Dimensional Smoulder Waves

by
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**USA**

## ABSTRACT

An infinite slab of porous fuel can support a two-dimensional smoulder wave, and this defines a free-boundary problem of Stephan type. In the classical Stephan problem the interface between the two phases is always free to move, but in the smoulder context this is only true for those portions of the interface that have a temperature greater than the ignition temperature.

Thus the smoulder wave has a leading edge at which ignition is first achieved, and the speed with which this leading edge moves controls the shape of the free boundary. A small parameter is defined by the air/fuel volume stoichiometry and, as a consequence, the smoulder wave is "shallow", faciliting an analytical treatment.

# Monotonicity Formula and Uniform Estimates for $\dfrac{\partial u}{\partial t} - \Delta u = \beta_\varepsilon(u)$

by
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USA

We discuss the issue of finding optimal a priori estimates for solutions of the singular pertubation problem :

$$Hu = \beta_\varepsilon(u)$$

uniform in $\varepsilon$.

Here $Hu$ is a parabolic operator, (for instance $\Delta u - u_t$) and $\beta_\varepsilon(u)$ an approximation of Dirac's $\delta$ (i.e. $0 \le \beta_\varepsilon(u, x, t) \le 1/\varepsilon$, and $\beta_\varepsilon \equiv 0$ for $u \le 0$ or $u \ge \varepsilon$).

Optimal regularity is of course Lipschitz, since the limiting free boundary problem has a gradient jump accross the free boundary.

The estimate

$$\sup_{B_{1/2}\times(-1/2,0)} |\nabla u_\varepsilon| \le C \sup_{B_1\times(-1,0)} |u_\varepsilon|$$

is obtained by a combination of a "two adjacent domains" monotonicity formula, and a variation of Bernstein technique that pushes $max|\nabla u|$ out of the transition strip, $0 < u < \varepsilon$.

# Simulation and modeling of the response of a flame front to acoustic perturbations

by
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## ABSTRACT

The interaction between a flame and an acoustic perturbation is of fundamental as well as technological interest. This topic has been investigated extensively in the past in relation with problems of combustion instability. Studies have concerned simple plane laminar flames propagating in a tube or more complex situations where the flame is turbulent. It has been found that flames respond to acoustic perturbations in many different ways. Flames are generally quite succeptible to acoustic excitation and resonant coupling may occur in cases where the flame is confined. In some cases the interaction is parametric and the flame moves with a period which is equal to the double of the acoustic period. In other cases the flame responds at the same frequency as the incident perturbations. We describe in this paper a study of the dynamical response of a two-dimensional flame front submitted to acoustic perturbations impinging on the flame from the upstream side. Direct numerical simulations are used to study this problem. The transfer function of the flame is determined from sytematic calculations carried out for a range of frequencies. The main features obtained from direct simulation are then retrieved with a kinematic model.

# Dynamics of Plane Detonations

by
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## ABSTRACT

The detonation fronts are known to be cellular (Denisov and Troshin 1959). The cells size which is typically twenty to fifty times larger than the thickness of the Chapman-Jouguet (CJ) detonation, was correlated with the ratio of the critical initiation energy of spherical detonations to the heat release per unit volume; the best phenomenological fits beeing obtained with 13/4 the size cells (Lee 1984). This yields the large numerical factor, $10^6$ - $10^8$, experimentally observed between the critical energy and the energy scale based on the CJ detonation thickness. In a recent analytical work we have shown that this factor results indeed from a nonlinear curvature effect amplified by a high temperature sensitivity (He et Clavin 1994). Correlations with the cell size seem fortuitous. The initiation conditions are the same for mixtures in which detonation fronts have no cell structure, as it is more likely the case for hot hydrogen-air mixtures encountered in safety problems of some nuclear plants. A clarification of the physical mechanisms responsible for the cell structures is required to provide a better insight into the problem. Despite the numerous studies since the pioneering work of Erpenbeck (1963), these mechanisms are not yet clearly understood. A longitudinal oscillatory instability of plane detonations was observed experimentally by Albert and Toong (1972) and by Lehr (1972), but preliminary results were first obtained at N.B.S. in 1962, see discussions p 476 in Ninth Symposium on Combustion. This 1-D instability was also described numerically first by Fickett and Wood (1966). As suggested by Abouseif and Toong (1986) the multidimensionnal patterns may result from a coupling between the longitudinal instability and transverse pressure waves propagating downstream the shock.

The purpose of the present analytic work is to clarify the physical mechanisms of the oscillatory instability and to provide a nonlinear description of the dynamics of unstable plane detonations. Consequences on detonation cells are briefly discussed. The details of the analysis and an comprehensive presentation of the results will be found in Clavin and He 1995. The analysis is carried out in the limit of an infinitely large temperature sensitivity of the induction time (5a). In order to avoid the spurious singularities of the square-wave model (Erpenbeck 1963), the temperture sensitivity of the rate of heat release is assumed to be smaller than the one of the induction process, (6a). Two further realistic approximations simplify the problem : i) A small difference in the heat capacities (5b) yields an evolution of the spatial distributions of temperature and heat release which are slaved to the temperature fluctuations of the inert shocked gases at the downstream shock boundary, independentaly on the pressure waves propagating across the induction and the exothermic regions; ii) The smallness of the shock velocity fluctuations (5c) allow to neglect the convective effects of the gas velocity fluctuations

on the entropy waves, see (6 b-c). Due to the high temperature sensitivity (5a), strong nonlinear effects are picked up even for such small fluctuations of the Neumann temperature $T_N$. Then, the nonlinear detonation dynamics reduces to slove two hyperbolic equations (7 a-b). For strongly overdriven detonations, the plane dynamics is fully controlled by a quasi-isobaric mass conservation leading to a nonlinear integral equation (10a) for the evolution of the front velocity. The density profile is a S-shaped curve with an inflexion point whose position relative to the shock depends on the shock velocity (incoming mass flux) but at an earlier time; the delay beeing associated with the propagation of entropy waves. This equation exhibits Hopf bifurcations whish represent the onset of galloping detonations. Comparisons with direct numerical simulations of plane detonations show a good agreement with analytical results (10a) even for strongly nonlinear regimes.

# The Newtonian Limit in Combustion Asymptotics

by
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## ABSTRACT

Enormous advances have been achieved in the theoretical understanding of combustion processes through the systematic and widespread use of asymptotic methods based on large activation energy. However, as attention has turned to the study of spatially non-uniform configurations, a very severe difficulty has been encountered. If the pressure, density, velocity and temperature in a gas are expanded asymptotically in terms of gauge functions, with the inverse activation energy as the small parameter, then one gets the equations of linear acoustics at leading order, with velocity and pressure fields driven by a temperature source. The equation for the first approximation to temperature perturbation is, however, the formidable transcendentally nonlinear wave equation

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\frac{\partial T}{\partial t} + \left(\frac{\partial^2}{\partial t^2} - \gamma^{-1}\frac{\partial^2}{\partial x^2}\right)\exp T = 0$$

first given by Clarke. Most approaches have then been forced to rely on a numerical solution of Clarke's equation at leading order, and the corresponding pressure, density and velocity fields then inferred. It is extremely difficult, however, to follow the expansions to higher order, or to diagnose their nonuniformities and the forms of appropriate matching expansions, because of the highly intractable problem posed at leading order by Clarke's equation.

This talk will review progress in an approach, due to P.A. Blythe and the author, in which a further asymptotic limit is taken, namely the "Newtonian" limit $\gamma \to 1$, $\gamma$ being the specific heat ratio. This allows Clarke's equation to be solved in the leading approximation, the space dependence appearing just parametrically. We show that the approach, when carried to higher approximations, gives not only valuable insight into the physical processes involved, but gives extraordinarily accurate results when compared with the available numerical solutions. Two particular problems will be studied: first, the problem of the motion in a reacting gas initiated by the motion from rest of a piston in a tube, and second, the development of the aerothermal fields in a one-dimensional box of gas in which there is an initially nonuniform temperature distribution.

Results for the induction phase in the first problem have been published (Blythe, P.A. & Crighton, D.G. *Proc. R. Soc. Lond. A* **426**, 189–209, 1989); we expect to publish details of the later stages of the evolution of the piston-generated explosion in a second paper, and in a third to give details of the evolution from an initially non-uniform temperature distribution.

# Inertia of a Propagating Flame

by
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## ABSTRACT

When seen as an interface propagating at a low Mach number, a flame behaves as a moving boundary which converts a fluid of one density into a fluid of another density. If it did not actually move through the fluid ahead of it, the flame would be analogous to the interface found at an air-water surface or at an ocean thermocline; the movement of such an interface involves shear between the fluids. However, because it propagates, the flame causes all shear that it produces to be shed into the burnt fluid as vorticity. As a result, the field of vorticity behind the flame carries a "memory" of the flame's earlier movement, which is analogous to the inertial effect of shear at a non-propagating interface.

Along with the action of acceleration, such as gravity or an overall oscillatory motion (for example from long wavelength acoustic disturbances), the movement of a flame thus contains features of both interfacial waves and of front-propagation. For example, in a gravitational field, flames are able to exhibit propagating or standing waves which are damped by propagation, as well as the Rayleigh-Taylor instability, the growth of which is limited by propagation.

The action of the inertial effect also accounts for behaviour peculiar to flames, such as the inversion of overextended tulip-flames. In an acoustic field, this inversion process can resonate with the interfacial-wave aspects of the flame's movement to create large amplitude oscillations of the flame-front that, depending on amplitude, frequency and wavelength, can mimic a parametric oscillator. Because flames are also sources of volume, this provides a strong mechanism through which flames may excite acoustic disturbances and vice-versa. The behaviour is, however, more complicated. Depending on initial conditions, acoustic accelerations of the same frequency can either create large flame oscillations or result in very little distortion of a flame, much less than the natural hydrodynamic distortion that a flame would experience without any imposed field of acceleration.

The presentation will explore the origins of these and other hydrodynamic properties of flames, viewed most simply as propagating interfaces between incompressible Euler fluids.

# Global Continuation and Complete Blow-Up for Quasilinear Heat Equations in Several Space Dimensions

by
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## ABSTRACT

In this joint research with J.L. Vazquez the possible continuation of solutions of the nonlinear heat equation in $R^N \times R_+$

$$u_t = \Delta u^m + u^p \quad \text{with} \quad m > 0,\ p > 1,$$

after the blow-up time is studied and the different continuation modes are discussed in terms of the exponents $m$ and $p$. Thus, for $m + p \leq 2$ we find a phenomenon of nontrivial continuation where the region $\{x : u(x,t) = \infty\}$ is bounded and propagates with finite speed. This we call incomplete blow-up. In particular, in the limit case $p + m = 2$ in 1D, we show that the singular interfaces (free boundaries) move with the minimal speed $2\sqrt{m}$ for large times, and in general are not $C^2$-functions.

For $N \geq 3$ and $p > m(N+2)/(N-2)$ we find solutions which blow-up at finite $t = T$ and become again bounded for $t > T$. Otherwise we find that blow-up is complete for a wide class of initial data. In the analysis of the behavior for large $p$ a list of critical exponents appears whose role is described. We also discuss a number of related problems and equations.

Our study concerns in all cases a uniquely defined class of proper solutions (or viscosity solutions) which are globally defined in time, though they may take infinite values.

We apply the same technique of analysis to the problem of continuation after the onset of extinction e.g. for the equation

$$u_t = \Delta u^m - u^p, \quad m > 0.$$

We find that no continuation exists if $p + m \leq 0$ (complete extinction), and there exists a nontrivial continuation if $p + m > 0$ (incomplete extinction when free boundaries occur).

# Bifurcation and Related Properties of Cellular Bifurcation of Travelling Waves in a Flame Propagation Model

by
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## ABSTRACT

We study in a mathematical framework the cellular bifurcation of the planar wave solutions of the thermo-diffusive model for flame propagation with high activation energies. In particular, we make precise the local dynamics of the flow and the different behaviors of the bifurcation in the three-dimensional case.

# Simulation of Preferential-Diffusion Effects in Jet Flames

by
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and

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## ABSTRACT

A vertically mounted jet diffusion flame formed between coannulus fuel and air jets provides a controlled system for studies on vortex-flame interactions. When the annulus air flow is low, buoyancy-influenced toroidal vortices form outside the flame surface. On the over hand, at a sufficiently high fuel jet velocity, the shear-layer instabilities begin to grow and vortices form inside the flame surface. As these inner and outer vortices convect downstream, they interact with the flame and create locally stretched and compressed flamelets and, thereby, form wrinkles on the flame surface. Earlier esperiments and theoretical studies on a low-speed $H_2$-air jet diffusion flame suggest that small pertubations roll-up into outer vortices due to buoyancy-induced instability and make the flame flicker at a frequency of 14 Hz. The experimental studies further indicate that the flame temperature tends to increase when the flame is compressed and decrease when it is stretched by the outer vortex.

Direct numerical simulations of the vortex-flame interactions in jet diffusion flames are performed using a time-dependent mathematical model incorporating buoyancy, finite-rate detailed chemistry, and transport coefficients that depend on temperature and species concentration. The effects of finite-rate chemistry and preferential diffusion during the vortex-flame interaction are discussed in this presentation. It is found that the non-unity Lewis number inside the flame–not the change in local Damkohler number–is responsible for the experimentally observed fluctuations in the flame temperature. It is also observed that preferential mass diffusion of different species causes an increase in water in the compressed regions of the flame and an increase in radicals in the stretched regions. Production of NO in the flame zone appears to be extremely sensitive to flame stretching and compression. The concentration of NO is found to increase significantly in the compressed flamelet that is formed during the outside vortex-flame interaction.

The effects of non-unity Lewis number in a premixed, $H_2 - O_2 - N_2$ flame are also studied by performing detailed calculations for different flow conditions. The temperature, species concentration, and velocity fields are investigated under fuel-lean, stoichiometric, and fuel-rich conditions. The calculations show that under fuel-lean conditions, the flame exhibits the "tip-opening", while under fuel-rich conditions, the tip of the flame burns intensily. Calculations also show that dilution of the fuel jet with nitrogen or argon causes the flame to burn intensely

along the sides, while dilution with helium causes the tip of the flame to burn intensily. These results are in agreement with experimental findings and confirm the fact that the tip-opening phenomenom results from the differences in heat- and mass-transport rates. In premixed flames, large outer vortices are found to form when the fuel is diluted with either nitrogen or argon. However, the dynamic characteristics of these vortices are quite different from those observed in jet diffusion flames. The vortex-crossing frequency is found to depend on the gas used as the diluent and its concentration.

# A Capturing/Tracking Hybrid Method for Premixed Combustion Simulations

by
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## ABSTRACT

A new flame front tracking algorithm for high-speed premixed combustion that is compatible with any modern shock capturing scheme for gasdynamics computations will be presented. The scheme relies on time updates of conserved quantities only, i.e., a separate storage of pre- and post-flame states in grid cells cut by the discontinuity is unnecessary as in a *capturing* method. For the evaluation of flame speeds accurate approximations of burnt and unburnt states are nevertheless required. These are obtained from the cell-averages by a reconstruction algorithm that uses the precise current flame location, conservation of mass, momentum, energy and species and the Hugoniot jump conditions. The flame front is *tracked* as the levelset of a suitable scalar field, which makes complex topological changes of the flame geometry accessible in a straight-forward fashion. One- dimensional results for one possible scenario of deflagration-to-detonation transition as well as two-dimensional computations of premixed flame stability and (partially) premixed high-speed combustion will demonstrate the current state of the development.

# Flame Development from a Hot Spot

by
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## ABSTRACT

A simple one-dimensional reactive-diffusive model for temperature and concentration during ignition is

$$u_t = u_{xx} + f(u)g(v), \quad v_t = av_{xx} - \varepsilon f(u)g(v).$$

It is easiest to assume that $a = 1$ so that the equations decouple and, with appropriate initial and boundary conditions, the model reduces to

$$u_t = u_{xx} + f(u)g(1 - \varepsilon u).$$

Such problems are known to have flame-like solutions and can alternatively be thought of as free-boundary problems with the thin reaction region (the flame) being replaced by a moving boundary.

Solutions are to be bounded, above and below, to show how flames form at a hot spot (where the solution to the model without reactant consumption, $u_t = u_{xx} + f(u)$, has become large). At present only the prototype problem where $g(v) = H(v)$, which gives a free-boundary problem directly, is being examined.

# The Analysis of Diffusion Flames with Non-Unity Lewis Number

by
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## ABSTRACT

In the description of the temperature, concentration and velocity fields in low Mach number flows with diffusion controlled combustion, following an overall irreversible reaction, we encounter jumps in the gradients of these flow variables at a reaction sheet, or flame surface, whose location must be determined together with the flow field. The numerical and analytical description of these flows can be simplified by the introduction of generalized Shvab-Zeldovich variables, or linear combinations of the temperature and concentrations, which satisfy a system of conservation equations free from concentrated sources at the flame sheet. A redefinition of the pressure and viscous stresses is also required to eliminate the difficulties introduced, in the numerical treatment of the momentum equation, by their jumps at the flame sheet.

# Pattern Formation and Spatiotemporal Dynamics in Combustion

by
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## ABSTRACT

We will discuss the results of both analytical and numerical studies of two models describing flames. We describe solutions exhibiting progressively higher degrees of spatiotemporal complexity, as parameters of the problem are varied.

# Heterogeneous Combustion Processes and Surface Phase Transitions

by
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## ABSTRACT

Starting from the model case of a decomposition process in some binary fluid mixture, the concept of nonlinear generalized diffusion is developed, given in terms of a quasi-linear degenerate ('weak') parabolic PDE. It is then shown that modelling of the free-surface -type problem of a retracting or spreading surface film is possible in terms of such a generalized diffusion process (assuming microscopic and mesoscopic film thicknesses). Due to the large disparity in the size of film thickness and film extension in lateral directions, the thin-film-approximation applies which is shown to bring about a separation of the effects of lateral film flow processes and transversal transport processes describing the formation as well the removal of film matter due to heterogeneous reaction and vaporization. For infinite films of thicknesses in the microscopic and mesoscopic range, the film evolution is described (in lowest order) by a film evolution equation, with surface phases corresponding to stationary films of constant thickness. Transitions between surface phases then are obtained in the form of 'sharp waves' (Barenblatt, Aronson, Samarskii), such that physical causality is satisfied. Since surface phase transitions may be seen as consisting in the spreading of one phase on top of the other, the occurrence of 'sharp' waves implies the existence of a 'true' contact angle.

# Multi-Dimensional Transition Layers in an Exothermic Reaction Diffusion Equation

by
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## ABSTRACT

Taking $\Omega_L$ as a cylindrical domain with the cross section $\Omega$ in $\mathbb{R}^n$ and the length $L$, that is $\Omega_L = \{(x,y) \in \mathbb{R}^{n+1}/0 < x < L, y = (y_1, y_2, \dots, y_n) \in \Omega\}$, we consider the following reaction-diffusion system :

$$
(1) \qquad \begin{aligned} &T_t = d_T(\Delta_y T + T_{xx}) + qk_l(T)a \\ &\qquad t > 0, (x,y) \in \Omega_L = \{(x,y) \in \mathbb{R}^{n+1}/0 < x < L, y \in \Omega\} \\ &a_t = d_a(\Delta_y a + a_{xx}) - k_l(T)a, \end{aligned}
$$

where $\Delta_y$ is the Laplace operator in y. $T$ is the temperature and $a$ is the concentration of the chemical reactant. The boundary conditions are :

$$
(2) \qquad \begin{aligned} &(T,a)(x,y,t) = (T^*, 0) \\ &\qquad t > 0, (x,y) \in \Gamma_i = \{(x,y) \in \mathbb{R}^{n+1}/x = iL, y \in \Omega (i = 0, 1)\}, \\ &T = T^*, d_a \partial a/\partial\sigma = \eta(a^* - a), \\ &\qquad t > 0, (x,y) \in \Gamma = \{(x,y) \in \mathbb{R}^{n+1}/0 < x < L, y \in \partial\Omega\}, \end{aligned}
$$

where $T^*$ is the given temperature on the boundary and $\sigma$ is the outerward normal unit vector on the boundary $\partial\Omega$. Th conditions (2) indicate that the temperature is fixed to be $T^*$ on the boundary, while the reactant flows into or out through the boundary $\Gamma$, where $\eta$ is the flux rate and $a^*$ is the given concentration on the boundary. $\eta$ and $a^*$ are both nonnegative constants but may depend on $x$ and $y$. In particular, the case when $\eta = 0$ means no supply of the reactant through the boundary $\Gamma$ (the system is closed), so that we can expect that $(T, a)$ tends to $(T^*, 0)$, because the reactant is totally consumed. When $\eta$ and $a^*$ are nonnegative but not identically zero, (1) with (2) is an open system, in other word, it is a far from equilibrium system.

With a suitable normalization for $(T, a)$, the problem (1),(2) is rewritten as the following system with new variables $(u, v)$ which denote respectively the temperature and the concentration of the reactant :

$$
(3) \qquad \begin{aligned} &u_t = \Delta_y u + u_{xx}) + vf(u) \\ &\qquad t > 0, (x,y) \in \Omega_L = \{(x,y) \in \mathbb{R}^{n+1}/0 < x < L, y \in \Omega\} \\ &v_t = d(\Delta_y v + v_{xx}) - \varepsilon vf(u) \end{aligned}
$$

with $f(u) = exp(u/(l + u/\alpha))$ for some constant $\alpha$ which approximately takes 5 -100. We assume that $\varepsilon > 0$ is a sufficient small parameter. The corresponding boundary conditions are :

$$(4) \quad \begin{cases} (u, v)(t, x, y) = (0, 0) \\ \qquad t > 0, (x, y) \in \Gamma_i = \{(x, y) \in \mathbb{R}^{n+1}/x = iL, y \in \Omega(i = 0, 1)\}. \\ u = 0, d\ \partial v/\partial\sigma = \varepsilon h(v^* - v) \\ \qquad t > 0, (x, y) \in \Gamma = \{(x, y) \in \mathbb{R}^{n+1}/0 < x < L, y \in \partial\Omega\}, \end{cases}$$

where $h$ and $v^*$ are nonnegative but not identically zero.

The purpose of this lecture is to understand the dynamics of $(u, v)$, depending on the effect of the shape of $\Omega$ as well as the length $L$ and the magnitude of the given concentration of the reactant $v^*$ on the boundary, by using the singilar limit analysis as $\varepsilon \to 0$ and complementarily numerical simulations.

The results obtained here are a joint work with K. Sakamoto of Hirosshima University.

Reference

M. Mimura & K. Sakamoto ; Multi-dimensional transition layers for an exothermic reaction-diffusion system in long cylindrical domains.

# The Fast Reaction Limit for a Reaction-Diffusion System

by
**L.A. Peletier**
**Mathematish Instituut**
**Rijksuniversiteit Leiden**
**Al Leiden, The Netherlands**

## ABSTRACT

In this talk we shall discuss the penetration problem for a substance entering a solid material, which reacts with the substance. We shall discuss the fast reaction - slow diffusion limit and prove that in this limit the problem becomes a free boundary problem. For some reactions, the solution of the original problem has a free boundary for finite reaction rate $k$. We then discuss the limiting behaviour of this boundary as $k \to \infty$. We shall see that thanks to a scaling argument, the large reaction limit is closely related to the large time behaviour, so that we obtain tha latter as a byproduct.

This problem arose in the penetration of radio-labeled anti-bodies into tumorous tissue.

# Dynamics of a Free Boundary Model of Thermal Instability

by
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## ABSTRACT

For many regimes of solid-state combustion, dynamics of the deflagration front are determined by the subtle interaction between the energy release by the combustion process and energy dissipation by the media. Properties of a well-known mathematical model that captures basic mechanisms of this interaction will be discussed in the talk.

The simplest version of the model consists of the heat equation for the temperature in a semi-infinite domain, with two boundary conditions at the free boundary which represent the heat conservation and the kinetic relation between the temperature and the interface velocity. For a natural class of kinetic functions the model problem has been proved to be well-posed globally in time, with uniformly bounded solutions, and to undergo a Hopf bifurcation as a certain kinetic parameter varies. Numerical simulations on the model reveal an amazing variety of dynamical patterns, including sequences of period doubling bifurcations leading to chaos, infinite period bifurcations and Shilnikov type trajectories. The dynamical scenarios appear to be of finite-dimensional nature. We discuss low-dimensional approximations of the free boundary model (by as little as three ordinary differential equations) that mimic dynamics of the system extremely accurately. There are strong indications that the model possesses a finite-dimensional attractor.

The work reported in the talk is a joint work with Michael Frankel of Indiana University–Purdue University at Indianapolis.

# Qualitative Analysis of Solutions for Simple Models of Dynamic Combustion

by
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## ABSTRACT

The equations which describe the evolution of a combustible gas display a great variety of nonlinear phenomena, including in particular those leading to the formation of combustion waves. While deflagration waves can be modelled under suitable assumptions by reaction-diffusion systems, coupling between the fluid dynamics of the gas mixture and the chemical kinetics plays a fundamental role in the description of detonation waves. In the resulting mathematical models the qualitative properties of solutions are determined by the interplay between convection, diffusion and source terms, as well as by the presence of different characteristics time scales. Typical features of such solutions are the presence of shock waves, instability phenomena and the appearance of spurious solutions in numerical calculations.

The purpose of this talk is to discuss some recent rigorous results concerning simplified mathematical models, which capture some relevant difficulties among those mentioned above.

# Weakly Nonlinear Analysis of the Pulsating Instability of Flames

by
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## ABSTRACT

As the Lewis number is greater than a certain critical number, $L_c$, a premixed flame exhibits the so-called pulsating (or oscillatory) instability: a pair of transversal counterpropagating wavetrains appears at the flame front. For adiabatic thermodiffusive flames, $L_c$ is unrealistically large but heat losses and thermal expansion effects may significantly decrease $L_c$. A pair of amplitude equations describe the weakly nonlinear evolution of the wavetrains near the instability limit if the flame is anchored, while for freely propagating flames a third amplitude equation must be added. Those equations contain a large parameter, in the generic case when the group velocity is of order unity, and this makes the analysis somewhat subtle but more amenable to purely analytical treatment. Several recent results concerning anchored flames will be presented showing that a rich dynamic behavior must be expected at the onset of this instability. Namely, quasi-periodicity, intermittency, period-doubing sequences and crises (i.e., collisions of limit cycles involving chaotic behavior) associated with symmetry gaining.

# Free Boundary Problems in Nonpreximed Combustion

by
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## ABSTRACT

Most of the free boundary problems in combustion concern premixed systems in which reaction fronts move in unbounded or partially bounded media. There are, however, certain problems of combustion for nonpremixed systems in which free boundaries arise. The presentation will review such problems, which typically involve combustion instabilities. Attention will be placed in particular on the diffusive-thermal instability of Linan's diffusion-flame regime for counterflow diffusion flames. The existence of such an instability, leading to cellular diffusion flames, will be demonstrated for Lewis numbers of limiting reactants less than unity. The approach reveals divergences with conventional scalings and involves construction of a composite dispersion relation from a composite expansion of dispersion relations derived from analyses of two distinguished limits. Correspondences are made with experimentally observed nonplanar patterns of diffusion flames near limiting conditions.